Applicability of Tank Model in Mid-Sized Catchments in Eastern Uganda

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Abstract Long-term stream discharge data is indispensable in irrigation and drainage design. However, in Uganda, this data is poor and insufficient, limiting irrigation system design. Conversely, the rainfall monitoring network is denser than the river flow monitoring network. Therefore, we attempt to build a model that calculates river discharge from input of rainfall. In this study, the lumped parameter Tank Model was applied. The model was applied to the Namatala River catchment (155 km^2) in Eastern Uganda. The study sought to ascertain the applicability of a lumped parameter model to a mid-sized catchment. Specifically, the objectives were: 1) To calibrate the numerical values of Tank Model parameters, 2) To verify the Tank Model parameters. This Tank Model required daily rainfall, evapotranspiration and river discharge data during calibration. Data for years 2015 and 2016 was used for calibration and validation respectively. During calibration, Monte Carlo simulation was used to find the numerical values of 16 Tank Model parameters. The best performing calibration parameter set had Nash-Sutcliffe (*NS*) efficiencies of 0.608 and 0.257 in calibration and validation respectively. However, among the 2015 calibration parameter sets, the one with a calibration *NS* of 0.502 performed best in validation (*NS* = 0.526). Equifinality was observed during parameter calibration. By using Tank Model, simulated discharge was divided into its surface runoff, interflow and base flow components. Tank Model was adaptable to Namatala River catchment.

Keywords Equifinality, lumped model, Monte Carlo simulation, parameter, Tank Model

INTRODUCTION

To buttress the vulnerable agricultural systems in Uganda, the government plans to promote irrigation (MWE, 2013). Irrigation and drainage planning requires long-term observed hydro-meteorological data. However, Uganda has a scarcity of this data. Numerous Ugandan rivers are ungauged, and worse, only about 33% of installed water level gauging stations are operational (MWE, 2013). Comparatively, the state of rainfall monitoring is better than that of the hydrological monitoring. Further, Kobayashi et al. (2018) explored the accuracy of satellite rainfall observations and found their detection accuracy to be acceptable. Satellite observed rainfall data is therefore a promising source of rainfall data.

Since rainfall data is more readily available, it is attempted to build a hydrological model to calculate river discharge from inputs of rainfall. Tank Model (Sugawara, 1995) and TOPMODEL (Beven and Kirkby, 1979), are some of the successful hydrological models. Okiria et al. (2019) applied the semi-distributed TOPMODEL to the Atari River catchment in Eastern Uganda. In this study, an attempt will be made to represent the hydrological response of the Namatala River catchment (NRc) using Tank Model. Whereas TOPMODEL is semi-distributed, and has fewer parameters, it is comparatively complex, and separates the predicted hydrograph into only two components, i.e., surface runoff and subsurface flow. On the other hand, Tank Model is simpler and separates the predicted hydrograph into four components, *i.e*., surface runoff, inter flow, sub-base flow and base flow. A weakness of Tank Model is that it is a lumped parameter model, with many unknown parameters (16 parameters for a 4-tank Tank Model). Because it is lumped, it cannot represent the heterogeneity in catchment characteristics. And due to the many unknown parameters, a more pronounced equifinality is expected. This equifinality might be caused by the interdependence among parameters reported in Beven (1997).

Whereas Tank Model has been successful in the continuously wet soils in humid sub-tropical climates (Suryoputro et al., 2017; Xiong et al., 2009), its behaviour in tropical climates, with soils that undergo both wet and dry periods, is not widely documented. Onyutha (2016) applied Tank Model to the Blue Nile Basin in Ethiopia. In East Africa, Tank Model was applied to a catchment in Rwanda (JICA, 2014). To date, the authors have not found published evidence of the application of Tank Model in Uganda. Therefore, the novelty of this study is to pioneer the development of Tank Model for a catchment in Eastern Uganda.

Before hydrological models are applied to ungauged catchments, they require calibration and validation. Model calibration involves the determination of unknown parameters and their predictive performance. On the other hand, validation involves testing the predictive performance of the calibrated parameters in a period other than that of calibration. Being one of the well gauged catchments in Uganda, the NRc was chosen for the calibration of Tank Model.

The lumped parameter model assumption of a catchment scale homogeneous hydrological response is likely to fail for medium to large catchments. Therefore, the purpose of the study is to ascertain the applicability of a lumped parameter model (Tank Model) to mid-sized catchments in Eastern Uganda. The specific objectives are: 1) Calibration of Tank Model parameters; and 2) Validation of the model parameters. In so doing, we strive to build a Tank Model that successfully predicts daily stream discharge from inputs of daily rainfall and evapotranspiration.

METHODOLOGY

Study Area

The study area is the Namatala River catchment (NRc), a headwater catchment of Mt. Elgon in Eastern Uganda, with a drainage area of 155 km^2 at the stream gauging station. Its topography is comprised of mountainous areas from where the Namatala River originates and flows to the relatively flat plains. From ASTER GDEM, the difference in height between the lowest and the highest point is 1,262 m. Of the 155 km^2, 73% is agriculture, 24% is forest, 2% is built up areas and 1% is rangeland.

Under the Project on Irrigation Scheme Development in Central and Eastern Uganda (PISD) (JICA, 2017), hydro-meteorological monitoring equipment were set up in the NRc, *viz*., a mid-stream rain-gauge to detect catchment rainfall, a downstream meteorological station to measure evapotranspiration parameters and a water level logger at a control section of the Namatala River.
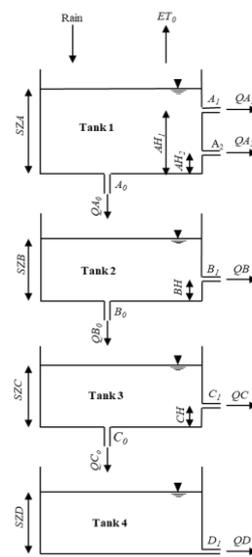
Tank Model Concept

One version of Tank Model comprises of four tanks laid out vertically in series, so named tanks 1, 2, 3 and 4. Rainfall is added to the topmost tank while evapotranspiration is subtracted from it. If tank 1 is empty, evapotranspiration is deducted from tank 2. If both tanks 1 and 2 are empty, then evapotranspiration is subtracted from tank 3 and so on. The storage tanks have side outlets, from

which runoff flows. Side outlets of tanks 1,2,3 and 4 release surface runoff, interflow, sub-base flow and base flow respectively. Tanks 1,2 and 3 have bottom outlets as well, through which infiltration to a lower tank occurs. Tank Model is calibrated to determine the value of 16 unknown parameters, namely; A_1 (coefficient of top side out let of tank 1), A_2 (coefficient of lower side outlet of tank 1), B_1 (coefficient of side outlet of tank 2), C_1 (coefficient of side outlet of tank 3), D_1 (coefficient of side outlet of tank 4), A_0 (coefficient of bottom outlet of tank 1), B_0 (coefficient of bottom outlet of tank 2), C_0 (coefficient of bottom outlet of tank 3), AH_1 (height of top side outlet of tank 1), AH_2 (height of lower side outlet of tank 1), BH (height of side outlet of tank 2), CH (height of side outlet of tank 3), SA_0 (initial height of water in tank 1), SB_0 (initial height of water in tank 2), SC_0 (initial height of water in tank 3) and SD_0 (initial height of water in tank 4), as in Fig. 2. The simulated river discharge is the sum of runoff from all the side outlets. Details of Tank Model are in Sugawara (1995).
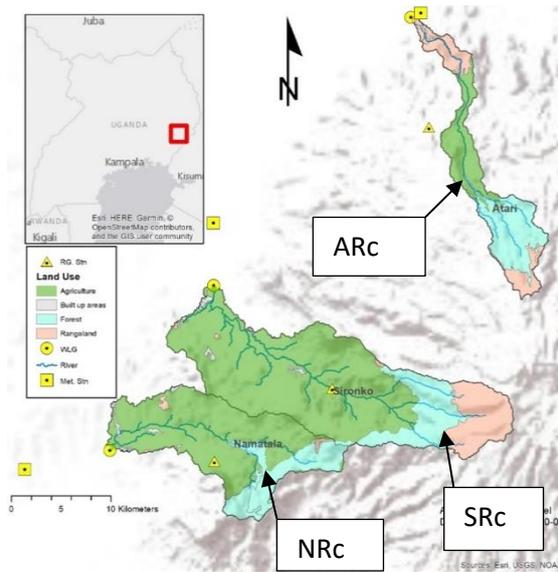
Fig. 1 Location of Namatala River catchment (NRc) relative to Atari River catchment (ARc) and Sironko River catchment (SRc)

Fig. 2 Schematic of Tank Model

Computational Procedure of Tank Model

Evaluation of tank 1:

$$SA_t = SZA_t + R_t - ET_{0t} \qquad (1)$$

SA_t is water depth in the tank during calculation (mm), SZA_t is water depth in the tank at the start and end of each time step calculation (mm), R is observed rainfall (mm/day), ET_{0t} is potential evapotranspiration (mm/day), and the subscript t is the time step of calculation (day).

$$QA_{xt} = \begin{cases} A_x \times (SZA - AH_x) \\ 0 \ for \ SZA < AH_x \end{cases} \qquad (2)$$

QA_{xt} is runoff from side outlet (mm/day), A_x is discharge co-efficient of side outlet, AH_x is height of side outlet (mm), and subscript x is side outlet under consideration (1 or 2).

$$QA_{0t} = A_0 \times SA_t \qquad (3)$$

QA_{0t} is infiltration from tank 1 to tank 2 (mm/day), A_0 is discharge co-efficient of tank 1 bottom outlet.

$$SZA_t = SA_t - QA_{1t} - QA_{2t} - QA_{0t} \qquad (4)$$

Evaluation of tank 2:

$$SB_0 = \begin{cases} SZB_0 + QA_{0t} - ET_{0t} & \text{for day 1, and an empty tank 1} \\ SZB_0 + QA_{0t} & \text{for day 1 and non} - \text{empty tank 1} \end{cases} \quad (5)$$

SB_0 is water depth in the tank during calculation (mm), SZB_0 is water depth in the tank at the start and end of each time step calculation (mm).

$$SB_t = \begin{cases} SZB_t + QA_{0t} - ET_{0t} & \text{after day 1, and an empty tank 1} \\ SZB_t + QA_{0t} & \text{after day 1 and non} - \text{empty tank 1} \end{cases} \quad (6)$$

SB_t is water depth in tank during calculation (mm), SZB_t is water depth in the tank at the start and end of calculation (mm).

$$QB_{1t} = \begin{cases} B_1 \times (SB_t - BH) \\ 0 \text{ for } SB_t < BH \end{cases} \quad (7)$$

QB_{1t} is runoff from tank 2 side outlet (mm/day), B_1 is discharge co-efficient of tank 2 side outlet, BH is height of tank 2 side outlet (mm).

$$QB_{0t} = B_0 \times SB_t \quad (8)$$

QB_{0t} is infiltration (mm/day) from tank 2 to tank 3, B_0 is discharge coefficient of tank 2 bottom outlet.

$$SZB_t = SB_t - QB_{1t} - QB_{0t} \quad (9)$$

The calculations for tanks 3 and 4 follow the same logic as the calculations for tank 2. Also, these conditions must be met: $A_0 > B_0 > C_0$ and $0 > C > 1$ where C is discharge coefficient.

Evaluation of Parameter Predictive Power

Nash and Sutcliffe efficiency (*NS*), (Nash and Sutcliffe, 1970) and Root Mean Square Error (*RMSE*) are the indices for evaluating model efficiency as in Okiria et al. (2019).

Novel Concept for Managing Competing Parameter Sets

Calibration of model parameters is an uncertain process with uncertainty increasing with increasing number of unknown parameters. The Generalised Likelihood Uncertainty Estimation (GLUE) framework was developed to eliminate competing parameter sets (Beven, 1997). To augment this concept, the authors suggested another method to manage uncertainty, *i.e.*: 1) Use the competing parameter sets to simulate the hydrograph for a period outside the calibration period (verification of competing parameter sets). The parameters that perform poorly in validation are then rejected; 2) Plot hydrograph components predicted using non-rejected parameter sets and choose the most likely hydrograph based on the depiction of the shape of the base flow component, which is supposed to be fairly constant. The authors attempted to use 1) and 2) in combination, to eliminate competing parameter sets.

Data Requirement

The input data for Tank Model is rainfall (*R*), river discharge (*Q*) and evapotranspiration (*ET*$_0$).

Table 1 Tank Model input data

| Purpose	Period	R (mm)	ET_0 (mm)	Q (mm)
Calibration	2015-Feb-27 to Oct-22 (238 days)	1,376	866	517
Validation	2016-Apr-27 to Dec-31(249 days)	1,004	921	467

RESULTS AND DISCUSSION

Like findings by Beven (1997), competing parameter sets were observed, making it impossible to identify with certainty the optimum parameter set for Tank Model.

Table 2 shows two parameter sets from the calibration in 2015 as well as their calibration and validation performances. During calibration, the best performing parameter set had an *NS* value of 0.608. However, during validation, its predictive power reduced to an *NS* of 0.257. On the other hand, the parameter set with the calibration *NS* of 0.502 had the best performance during validation, with a validation *NS* of 0.526. The difference between calibration and validation *NS* values could be evidence that parameter sets are dependent on rainfall characteristics as reported in Okiria *et al*. (2019). It can also be attributed to the obscurity in setting the initial condition during validation. Figure 3 shows the best performing Tank Model parameters. Based on the new method to manage uncertainty, the parameter set with calibration and validation *NS* of 0.608 and 0.257 respectively is preferred since it has a very good calibration *NS* and a sufficient validation *NS*, and it also yields a reasonable base flow curve – the classification of *NS* as good or sufficient is in Foglia et al. (2009).

Fig. 4 shows the observed and simulated hydrographs for 2015 during calibration while Fig.6 shows the observed and simulated hydrographs for 2016 during validation of 2015 parameters. In 2015, the trend of the simulated hydrograph was quite similar to that of the observed hydrograph. However, in 2016, the trends between observed and simulated hydrographs were less similar. The dissimilarity in 2016 could be attributed to input data error or in an ability of the model to accurately represent the rainfall-runoff response of 2016. In both years, majority of the peak discharges were underestimated. This could be attributed to under estimation of catchment rainfall and or the over estimation of evapotranspiration.

In Fig. 5 the components of the simulated hydrograph for 2015 during calibration are shown while Fig.7 shows the components of the simulated hydrograph for 2016 during validation by 2015 calibration parameters. The simulated discharge was separated into its surface runoff, inter flow, sub-surface flow and base flow components. Surface runoff was dominant during the rainy period while base flow was the dominant component in the dry period. In addition, base flow was generally stable and showed delayed response to rainfall.
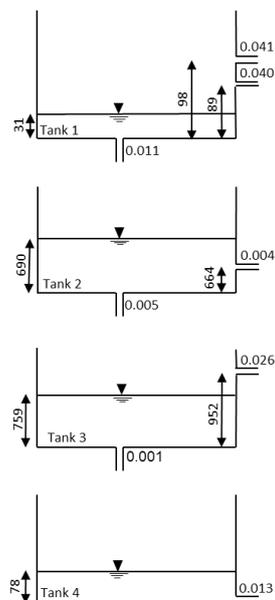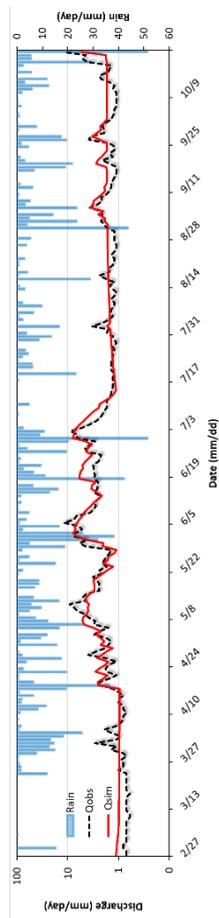

Fig. 3 Best performing parameters for Namatala River catchment (NRc) calibrated in 2015
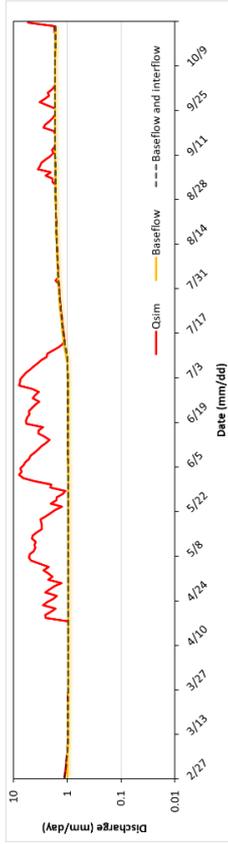
Table 2 Best parameter set from calibration in 2015 and their validation performance in 2016

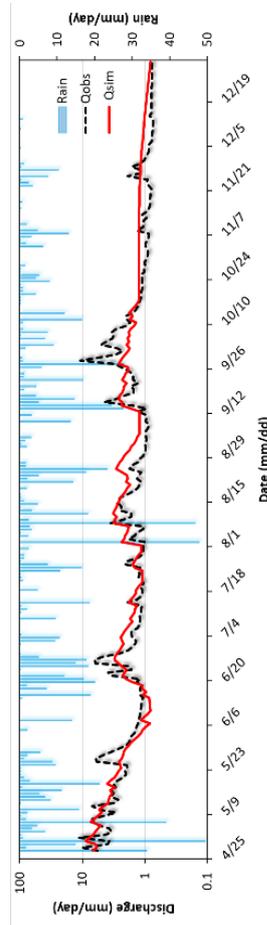| A_1	A_2	A_0	B_1	B_0	C_1	C_0	D_1	$SA_{initial}$	$SB_{initial}$	$SC_{initial}$	$SD_{initial}$	AH_1	AH_2	BH	CH	NS_c	$RMSE_c$	NS_v	$RMSE_v$
0.041	0.040	0.011	0.004	0.005	0.026	0.001	0.013	31	690	759	78	98	89	664	952	0.608	1.141	0.257	1.387
0.028	0.008	0.010	0.002	0.006	0.069	0.004	0.005	124	100	528	51	63	7	55	502	0.502	1.285	0.526	1.108

Note: NS_c and $RMSE_c$ are NS and RMSE values respectively during calibration, and NS_v and $RMSE_v$ are NS and RMSE values respectively during validation

Fig. 5 Hydrograph components for 2015 during calibration

Fig. 7 2016 Hydrograph components during validation by 2015

Fig. 4 Observed and simulated hydrograph for best parameter of 2015

Fig. 6 2016 Observed and simulated hydrograph using 2015 parameters

Note: Q_{obs} is observed discharge, Q_{sim} is simulated discharge

CONCLUSION

Tank Model was successfully applied to the Namatala River catchment to simulate its rainfall-runoff process. However, due to equifinality, the model should be applied with caution. In addition, more research needs to be done to better understand the shortcomings of the model. Acquisition of finer spatial resolution input data is recommended. Studies on other catchments in Eastern Uganda could be useful in confirming the applicability of lumped parameter models.

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